C31A-0600: Inversion of FMCW radar with a priori information from SMP measurements

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Abstract

Ground based radars such as the Frequency Modulated Continuous Wave (FMCW), are used to help quantify the spatial variability across a study area. The FMCW can observe major layers within the snowpack and estimate snowpack height. This project uses inverse methods to determine the snow density as a function of depth for each FMCW radar trace with a priori snowpack information estimated from the Snow Micro Penetrometer (SMP). With this information, we can look at how the density distribution, and the associated layer thicknesses, are affecting backscatter values during remote sensing calibration and validation campaigns.

1. FMCW Theory

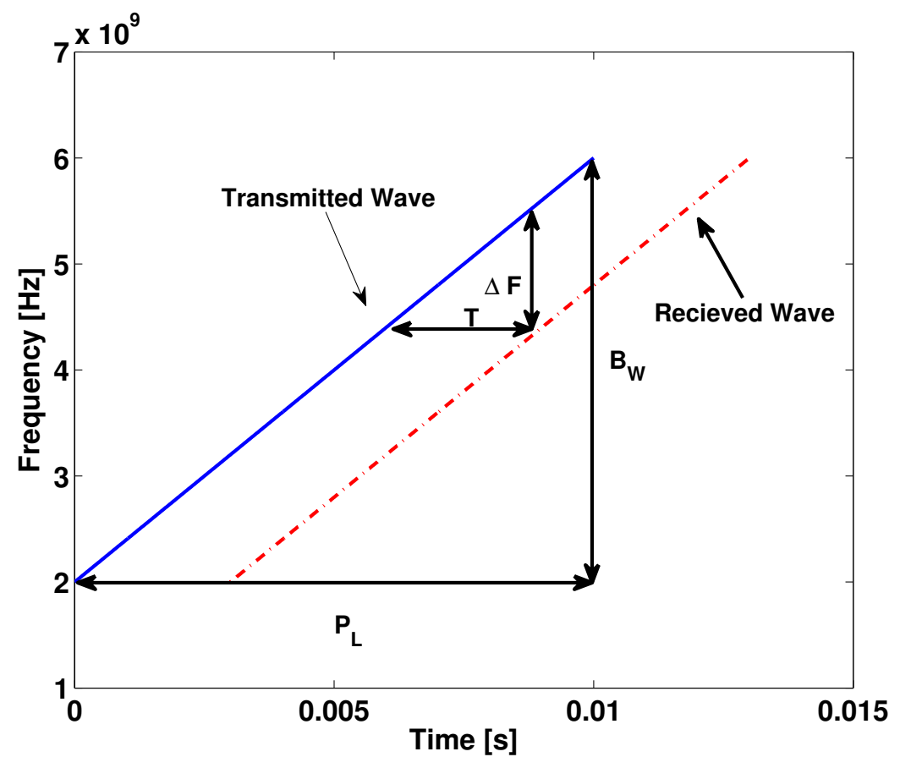

Figure 1: Frequency vs time for a FMCW transmitted (solid line) and received (dashed line) waveform. From [5]

- Frequency varies linearly with time between 2-10 GHz
- Measures an instantaneous frequency difference (Δf) of transmitted and received signals that is linearly related to two way travel time
- Signal peaks occur at each reflection, corresponding to a Δf and two way travel time
- Measures amplitude

2. Snow Micro Penetrometer (SMP)

- The SMP measures the penetration force with high-resolution (250 force measurements per mm with a 5mm diameter cone tip)
- Micro-structural and micro-mechanical properties can be estimated from the signal's geometry [3, 4].
- Layer heights can be determined to within 5mm [6]
- Density can be estimated from an empirical relationship with the penetration force

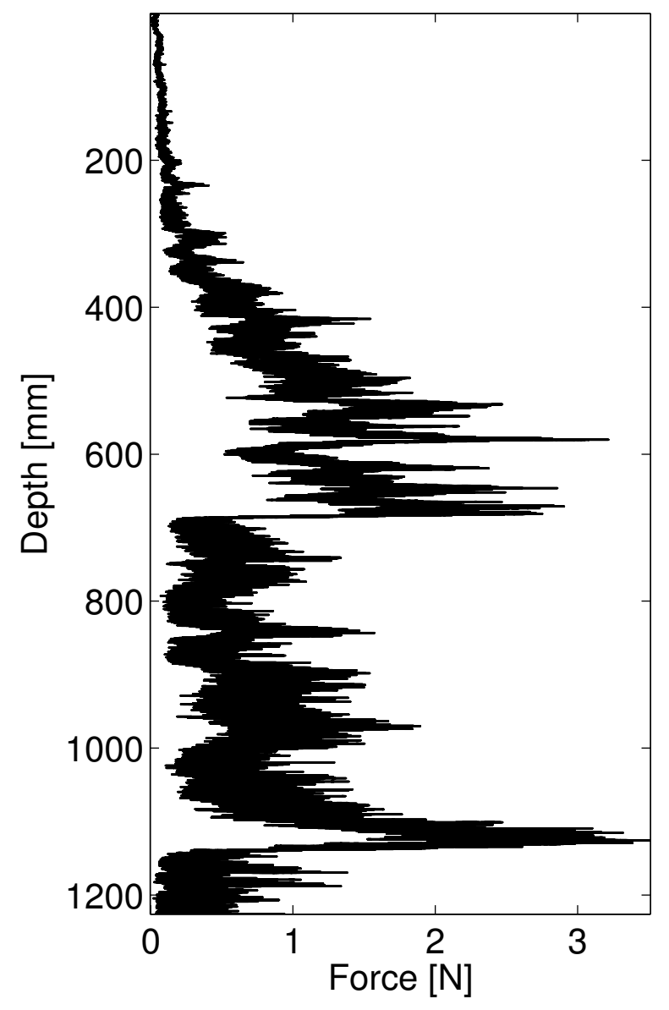

Figure 2: Example SMP profile from a remote sensing validation and calibration campaign in Colorado.

3. FMCW Forward Model

Figure 3 is a schematic for a multi-layer dielectric model. Each layer is described by its dielectric constant ϵ_j and thickness z_j. ϵ_{snow} is described with the complex refractive index method (CRIM) which assumes ϵ_{snow} is the volumetric sum of each component [8, 7]. Assuming a dry snowpack:

$$\epsilon_{snow,j} = [(1-\rho_{s,j})\sqrt{\epsilon_{air}} + \rho_{s,j}\sqrt{\epsilon_{ice}}]^2$$

with $\epsilon_{air} = 1$, $\epsilon_{ice} = 3.17$, $\rho_{s,j}$ is the snow density for each layer.

The reflection coefficient is calculated in an iterative process starting at the snow soil interface. The surface reflection coefficient is multiplied with the transmitted signal to calculate the received signal. This received signal is mixed with the transmitted signal, simulating the hardware within the FMCW. The magnitude of the modeled signal is equivalent to the PSD of the time domain FMCW signal. See Bradford and others (NS51A-1742, Friday) for more information.

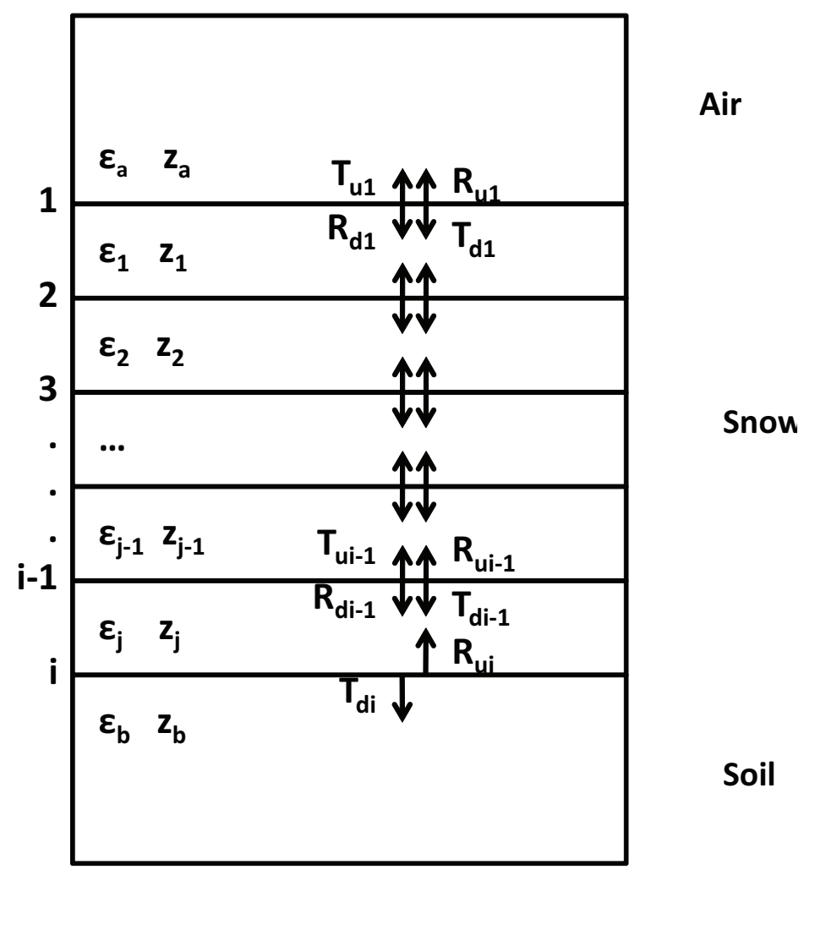

Figure 3: Schematic for a multi-layer dielectric model. The transmission and reflection coefficient direction is in reference to a downward wave propagating through the medium. Material properties are included in ϵ_j with layer thickness of z_j.

4. The Kalman Filter

The general FMCW inversion problem can be formulated as follows:

$$\mathbf{d}_{obs} = g(\mathbf{m}_{true}) + \mathbf{e}_{obs}$$

where $\mathbf{d}_{obs} \in \mathbb{R}^{kN\times 1}$ are observed radar measurements with k number of traces of length N. $\mathbf{m}_{true} \in \mathbb{R}^{k(2nl)\times 1}$ is the unknown snow parameters to be estimated at each radar trace with nl number of layers $m_k = log(\rho_1, ..., \rho_{nl}, z_1, ..., z_{nl})^T$. g is the non-linear model relating $\mathbf{m}_{true}$ to $\mathbf{d}_{obs}$; $\mathbf{e}_{obs}$ is random measurement noise with $N(0, \mathbf{C}_{obs})$. By adding lateral constraints in 2-D to parameters ($\mathbf{R}_p$) and a priori information from the SMP ($\mathbf{P}_{h-prior}$, $\mathbf{P}_{\rho-prior}$), the inversion problem becomes:

$$\begin{bmatrix} \mathbf{G} \\ \mathbf{P}_{h-prior} \\ \mathbf{P}_{\rho-prior} \\ \mathbf{R}_p \end{bmatrix} \cdot \delta\mathbf{m}_{true} = \begin{bmatrix} \delta\mathbf{d}_{obs} \\ \delta\mathbf{m}_{h-prior} \\ \delta\mathbf{m}_{\rho-prior} \\ \delta\mathbf{r}_p \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{obs} \\ \mathbf{e}_{h-prior} \\ \mathbf{e}_{\rho-prior} \\ \mathbf{e}_p \end{bmatrix}$$

or more simply:

$$\mathbf{G}' \cdot \delta\mathbf{m}_{true} = \delta\mathbf{d} + \mathbf{e}'$$

Snow parameters are estimated by the Kalman filter in an iterative process:

$$\begin{aligned} \mathbf{K}_i &= (\mathbf{G}'^T\mathbf{C}'^{-1}\mathbf{G}' + \mathbf{C}_{est,i})^{-1}\mathbf{G}'^T\mathbf{C}'^{-1} \\ \mathbf{m}_{i+1} &= \mathbf{m}_i + \mathbf{K}_i\delta\mathbf{d}_i \\ \mathbf{C}_{est,i+1} &= (\mathbf{I} - \mathbf{K}_i\mathbf{G}')\mathbf{C}_{est,i} \end{aligned}$$

$\mathbf{C}'$ is a diagonal matrix of error covariance matrices. $\mathbf{C}_{est,i}$ is the error covariance matrix of the current model estimate. $\mathbf{G}$ is the Jacobian matrix about $\mathbf{m}_i$. This extends work in [2] and [1] by iteratively updating the model error covariance matrix.

5. One Trace

A single trace, four layer model:

$$\mathbf{m}_{true} = (200, 250, 350, 250, 0.3, 0.3, 0.3, 0.3)^T$$

was used to help understand how the a priori information from the SMP in the initial guess $\mathbf{m}_0$, $\mathbf{P}_{h-prior}$, and $\mathbf{P}_{\rho-prior}$ affects the solution. If no a priori information from SMP is used (figure 4), the inversion cannot accurately capture the change in density.

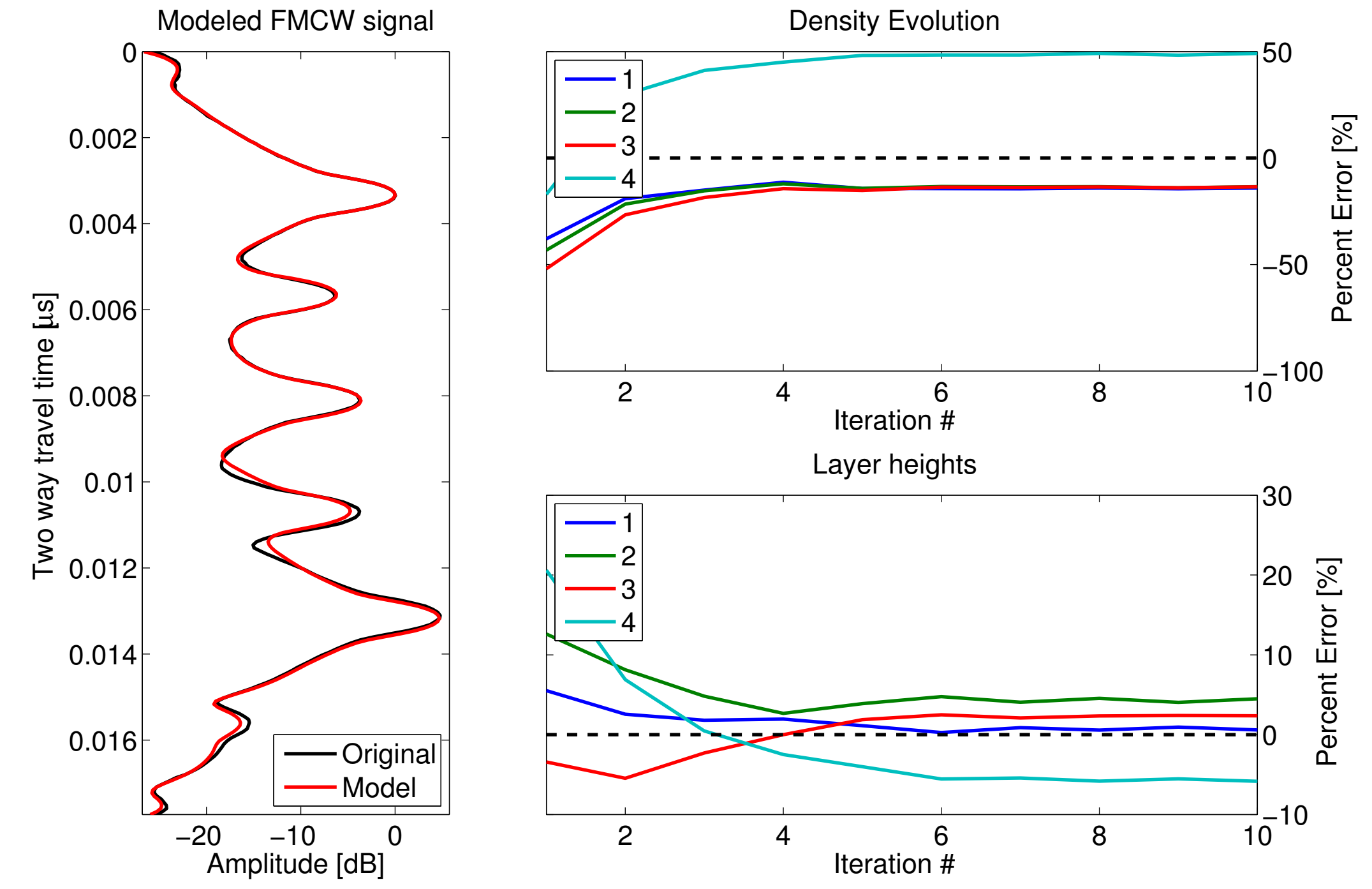

Figure 4: Density inversion with initial guess of $\mathbf{m}_0 = (57, 63, 68, 75, 0.35, 0.35, 0.35, 0.35)^T$. Some a priori information on the trend of density is needed for convergence.

If a priori information on layer heights ($\mathbf{P}_{h-prior}$) and initial densities ($\mathbf{P}_{\rho-prior}$) estimated from the SMP with an initial guess $\mathbf{m}_0$ that reflects the trend in density, the results converge to a robust solution (figure 5).

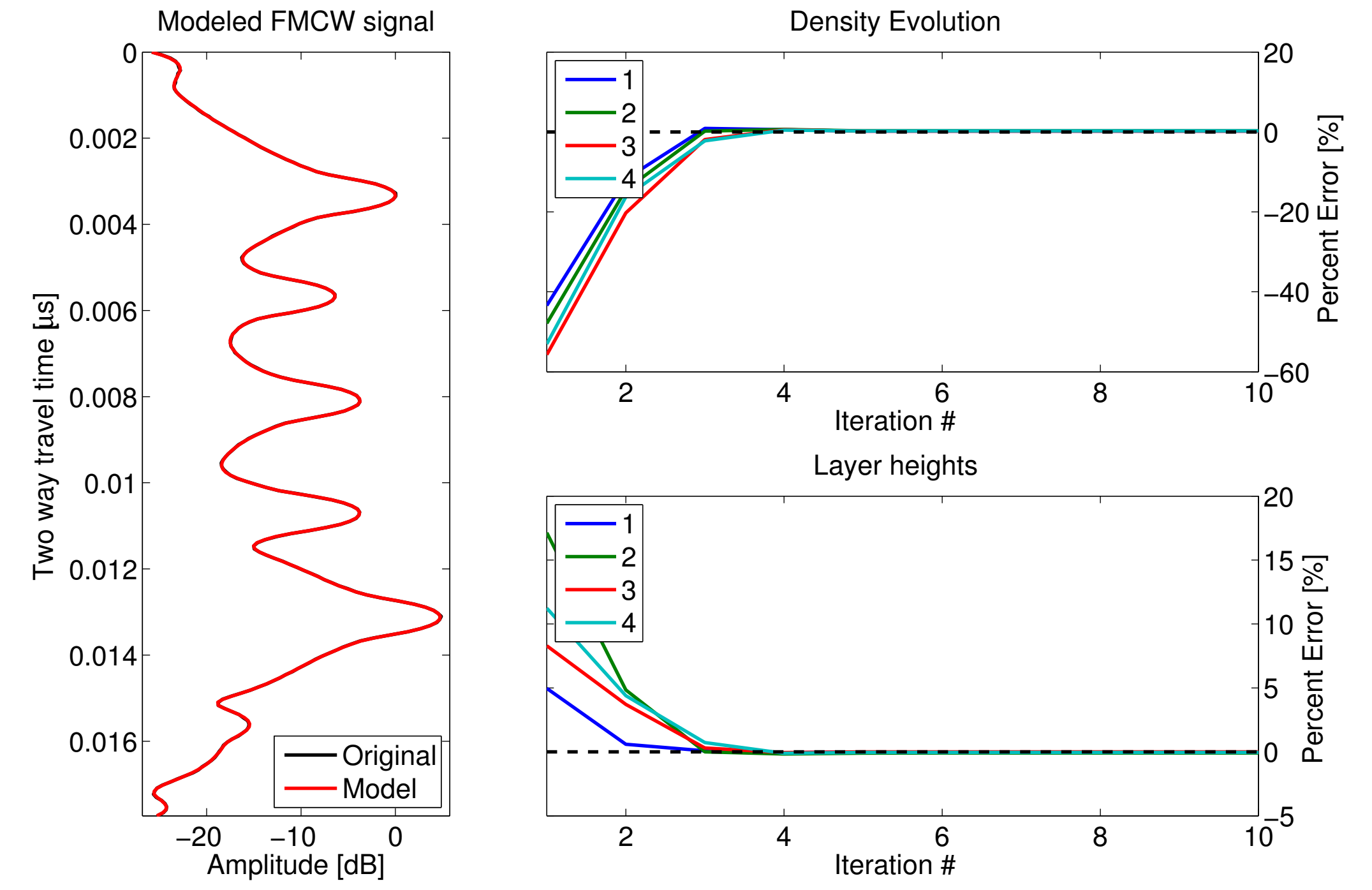

Figure 5: Density inversion using prior information from the SMP with an initial guess of $\mathbf{m}_0 = (57, 63, 68, 63, 0.35, 0.35, 0.35, 0.35)^T$.

6. Radar Transect

A synthetic FMCW radar transect with 50 traces in figure 6 shows the inversion using a priori information estimated from the SMP ($\mathbf{P}_{h-prior}$ and $\mathbf{P}_{\rho-prior}$) at trace 10 and 40, with lateral constraints on the snow parameters ($\mathbf{R}_p$). The initial guess $\mathbf{m}_0$ is derived from SMP measurements.

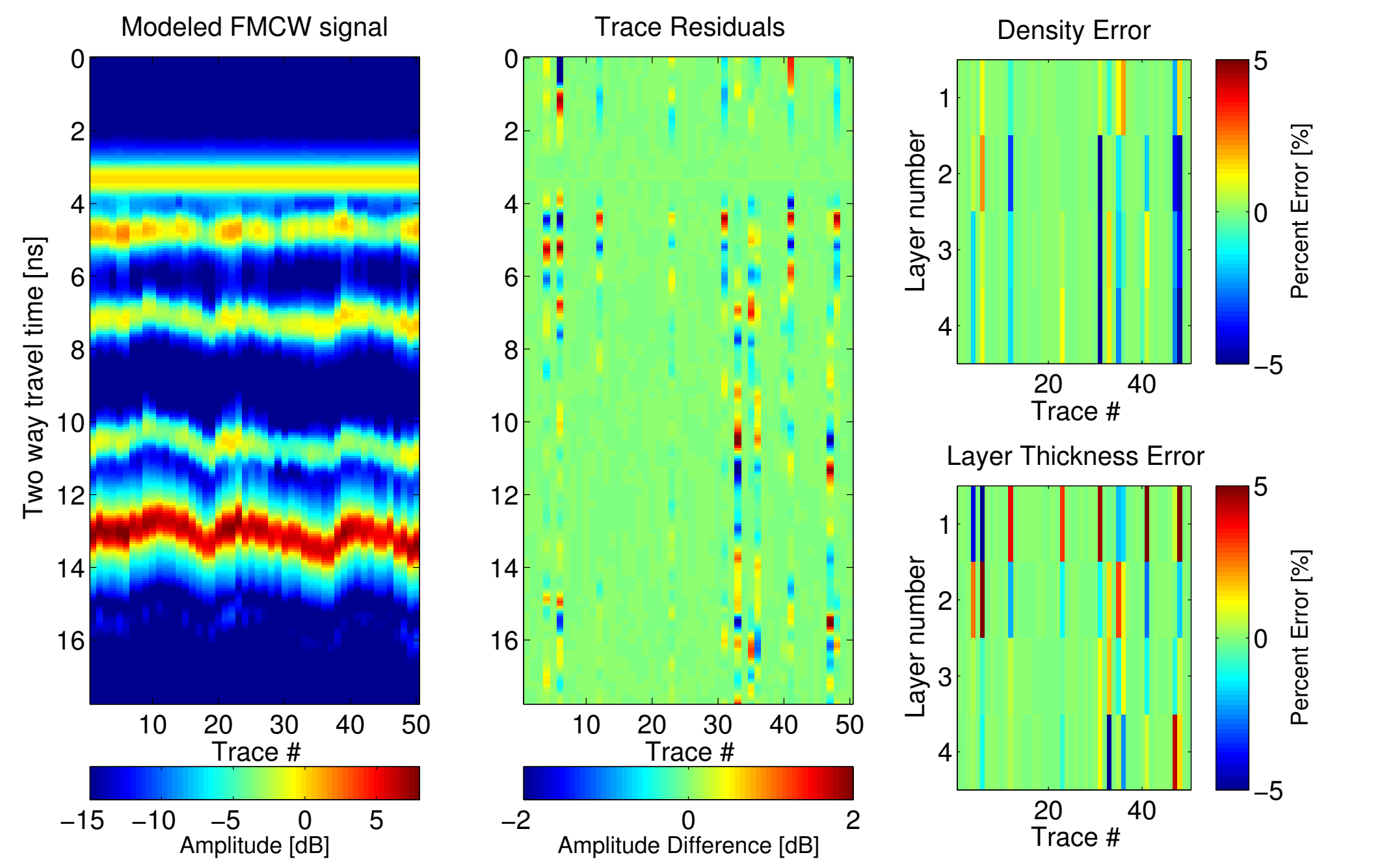

Figure 6: Density inversion of synthetic 4 layer FMCW transect using a priori information from the SMP at trace 10 and 40.

7. Conclusions

- A priori knowledge of the *trend* in density is needed (from SMP) for robust inversion
- Large horizontal changes in layer properties will be smoothed slightly, causing inversion errors
- A priori information from SMP, along with lateral constraints improves inversion convergence of density and layer height estimation, especially on transects
- Layer density and thickness can be determined for large transects to capture spatial variability

8. Future Work

- To use actual FMCW signals: identify source spectrum of instrument
- Weight higher amplitude signals for calculating residuals
- Use snow depth measurements as additional data set in between SMP measurements for long transects
- Determine how different layer thickness and density contrasts effect inversion
- Optimize field sampling strategy for SMP and snow depth

Acknowledgments

Work was funded by NASA Earth and Space Systems Science Graduate Fellowship (NNX10AF46H) and NASA grant Grand Mesa Ground Based Calibration (NNX10AU61G).

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