BEC Vortex Matter

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Outline

1. Bosons: what are they?
2. Bose-Einstein Condensation (BEC)
3. Vortex Formation: Rotating BECs
4. Current Work: theory, visualization, results
5. Acknowledgements
History of Bosons

• 1924 – Physicist S. Bose realized that classical particle statistics was insufficient for describing photons and developed a new statistics for photons

\[ f(E) = \frac{1}{e^{E/kT}} \]  

Boltzmann distribution becomes Bose-Einstein distribution!

• 1924 – A. Einstein generalized this formulation to massive particles (i.e., matter)
What is a Boson?

- **Bosons:**
  - examples: photons, $^4\text{He}$, $^{87}\text{Rb}$
  - integer spin (0,1,2,…)
  - prefer to be in the same quantum state
  …satisfy Bose-Einstein statistics

- **Fermions:**
  - examples: electrons, protons, neutrons
  - fractional spin (1/2,3/2,…)
  - cannot be in the same quantum state
  (Pauli exclusion principle)
  …satisfy Fermi-Dirac statistics

What is a BEC?

"From a certain temperature on, the molecules condense without attractive forces, that is, they accumulate at zero velocity. The theory is pretty but is there also some truth to it?" - Albert Einstein

- Matter waves have an associated de Broglie wavelength
  \[ \lambda_{dB} = \frac{h}{p} \]
- As matter cools, \( \lambda_{dB} \) grows
- When \( \lambda_{dB} \sim r_0 \), most atoms have same wave function
  ...BEC!

DeBroglie wavelength
\[ \lambda_{dB} = \frac{h}{\sqrt{2\pi m kT}} \]

At critical temperature and density, average distance between atoms becomes comparable to \( \lambda_{dB} \)

Result: Most atoms end up in same wavefunction
Is It Condensed?

- Velocity distribution as a dilute gas is cooled into the nano-Kelvin range.
- Distribution shows particles accumulate at zero velocity.
- Experimental proof of BEC!

www.maloka.org/f2000/bec/ what_it_looks_like.html
History of Bose-Einstein Condensates (BECs)

- 1924 – Bose and Einstein describe statistics of bosons
- 1932 – Keesom and Clusius measure the “λ-point” temperature of liquid $^4$He. Below this temperature, $^4$He becomes a superfluid.
- 1938 – London suggested an explanation for the properties of cold $^4$He: treat as an ideal gas obeying Bose-Einstein statistics (a cold aggregate of bosons in their lowest energy level).
- Today – Similar aggregates of cold bosons are studied using dilute atomic gases such as $^{87}$Rb, $^7$Li, and $^{23}$Na.

Phase diagram for $^4$He

R.Rothe, Bose-Einstein Condensation: A new kind of matter, 1995
History of BECs: The pioneers of modern BECs

Key steps to BEC with a cold gas of atoms:

- 1\textsuperscript{st} The development of the laser cooling process
  
  \url{http://www.colorado.edu/physics/2000/bec/index.html}

- 2\textsuperscript{nd} The creation of BECs in dilute gases of alkali atoms

1997 Chu, Cohen, Phillips

2001 Cornell, Ketterle, Weiman
Potential Application of BECs

- Atom Lasers
  - Quantum Lithography
  - Matter-Wave Inferometry
- Ultra precise gyroscopes
  - potentially $10^{10}$ times more accurate than laser gyroscopes
- Slow Light
  - Information Storage, Manipulation
- Quantum Computation?
  - BECs in optical lattices

Nature © Macmillan Publishers Ltd 1999
BEC apparatus

- Condensing dilute atomic gases at MIT: BECs are now a reality.

\(^{23}\text{Na setup @ MIT; W. Ketterle, et al.}\)
Trapping and Cooling the Substrate

• Magnetic Trap
  http://www.colorado.edu/physics/2000/bec/mag_trap.html

• Evaporative Cooling
  http://www.colorado.edu/physics/2000/bec/evap_cool.html
  Cornell, *Very Cold Indeed: The nanokelvin physics of Bose-Einstein condensates*, 1996

• Laser Cooling
  http://www.colorado.edu/physics/2000/bec/lascool1.html
Guinness World Record: $T < 0.5 \text{nK}$

Ketterle \textit{et al.}
Rotating BECs: Vorticity

- Strange property of BEC superfluids: they are irrotational.
- Large aggregates of vortices appear in the fluid, if we spin the BEC rapidly enough.

Spinning BECs \rightarrow Vortex Matter!

Vorticity in fluid flow

http://www.ldeo.columbia.edu/edu/dees/ees/climate/slides/vorticity.gif
How do you spin BECs?

With Lasers!

- Lasers are shone on the BEC, which knocks bosons out of the condensate.
- The radius of the condensate decreases, and the mass decreases.
- If the radius and mass of a spinning object decrease, that object will spin faster due to conservation of momentum:

\[ L = m \cdot r \cdot \omega \]

And with spinning traps!

- BEC is distorted, then the MOT is set to spin.
- Spinning MOT induces spinning BEC.
Spinning BECs: 2D Condensates

- As the BEC spins, it begins to “flatten out” due to centrifugal forces
- The higher the rotation rate becomes, the flatter the BEC gets
- Approximate our BEC as a 2D system $gN << \hbar\Omega_z$

Vortices: Stabilizing the Lattice

Vortex Matter!

- As the rate of spinning increases, the vortices stabilize, forming vortex arrays
- Triangular Abrikosov lattice
- Hundreds of vortices
Materials Science of Vortex Matter

- Unusual form of matter
  - Macroscopic quantum phenomena
  - Obeys Eulerian dynamics (moves perpendicular to $\vec{F}$)

- Areas of Study
  - Phases, Statics, Dynamics, Mean-Field theory
  - Quantum fluctuations, quantum melting, exotic states

- Comparison to experiment…(e.g. group of E.A. Cornell)
Rotating BEC’s – $H_{\text{eff}}$

How to treat a rotating system?:
go to the rotating frame

$$H_{\Omega} = H_{\text{lab}} - \Omega \cdot L$$

$$H_{\Omega}^0 = \frac{(p - m\Omega \hat{z} \times r)^2}{2m} + \frac{1}{2} m(\Omega_{\text{c}}^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2} m\Omega_{\text{z}}^2 z^2$$

$B_{\text{eff}}$ in z-dir with $\Omega_{\text{c}} = 2\Omega$

reduced radial confinement

Same as charged particles in a strong magnetic field!

Rapidly rotating limit $\Omega \rightarrow \Omega_R$
Landau Levels: 2D particles in a strong B field

Classical: cyclotron frequency

$$\Omega_C = \frac{eB}{m}$$

Quantum Mechanical: Landau levels

$$E_n = \hbar \Omega_C (n + \frac{1}{2})$$

Landau levels are macroscopically degenerate

$$gN << \hbar \Omega_C$$

$$n=0 \text{ LLL}$$

Lowest Landau Level approx.

$$z = x + iy$$

$$\phi(z) = \Phi_0 e^{-|z|^2/4} \prod_{i=1}^{N_V} (z - z_i)$$
Vortex Matter: 
Statics

- Equilibrium: \( \vec{F}_{\text{net}} = 0 \)

\[
\frac{E}{N} = \int d^2 z \left[ V(z) |\phi(z)|^2 + \frac{gN}{2} |\phi(z)|^4 \right]
\]

Trapping Term \hspace{2cm} Interaction Term

\[
f_i = - \frac{\partial}{\partial z_i^*} \left( \frac{E}{N} \right) = 0
\]

At equilibrium, the force at each vortex is zero.
Numerical Energy Minimization

- **Steepest Descent**
  - Robust, yet inefficient method
  - Step directions perpendicular to previous step
  \[ d_i \cdot d_j = 0 \]

- **Conjugate Gradient**
  - Step directions conjugate to previous step
  - Efficiently reaches local extremum in \( O(Nr) \) steps
  \[ d_i \cdot A \cdot d_j = 0 \]
Vortex Matter: Dynamics

- Second derivatives of the energy determine the restoring forces
- Action diagonalized using the Bogoliubov transformation
- Analogous to vortices connected by springs
- Eulerian dynamics: motion perpendicular to applied force
- For $N_v$ vortices, there are many numerically distinct physical modes of oscillation
eigenvalue(1): 0.00000

Vortex Dynamics: The Movie*
Starring: 37 Vortices in the Lowest Landau Level

Director & Producer: J. C. Díaz-Velez, Boise State Univ.
Executive Producer: C. B. Hanna, Boise State University

Based on calculations by
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Simulating vortex matter: An interactive approach via applets

- Meaningful comparisons to experiment can be made via an interactive simulation.
- Java applets are a convenient, interactive solution.

http://newton.boisestate.edu/~asup/VortexApplet.html

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