Instructions:

2. Work each problem on the exam booklet in the space provided.
3. Write neatly and clearly for partial credit. Cross out any material you do not want graded.

Name: _________________________________

Problem 1: _________________________/20
Problem 2: _________________________/25
Problem 3: _________________________/25
Problem 4: _________________________/30
Total: _________________________/100

Maxwell’s Equations:

Gauss’ Law: \( \phi_{in} = \phi_{out} \)

Ampere’s Law: \( \sum_{k} H_k l_k = I_{enclosed} \)

Faraday’s Law: \( v(t) = \frac{d\lambda}{dt} \)

Magnetic Circuits:

Ampere’s Law: \( N i = H l \) or \( F = N i = R \phi, \ R = \frac{1}{\mu A}, \ L = \frac{N^2}{R} \)

Faraday’s Law: \( v(t) = \frac{d\lambda}{dt}, \ \lambda = N \phi, \ \phi = BA, \ B = \mu H, \ \mu_o = 4\pi \times 10^{-7} \)
Problem 1 (20 Points)

Find the inductance of the N-turn coil. (Use $N = 100$ turns. The permeability of the magnetic core is assumed infinite and leakage and fringing effects are neglected.)
Problem 2 (25 Points)

The dimensions of the above magnetic structure are as follows:

Mean path length: $l = 25$ cm
Air-gap length: $g = \pi$ mm
Width: $w = 5$ cm
Depth: $d = 5$ cm
Number of turns: $N = 100$ turns

(a) Find the current $i$ needed to create a magnetic flux density $B_a = 1$ T in the air gap.
(b) Find the current $i$ needed to create a magnetic field intensity $H_c = 3750$ A-t/m in the core.
(c) If $i = 50$ A, find the magnetic field intensity $H_c$ A-t/m in the core.
The above magnetic structure with varying cross-section is made of a magnetic material with the B-H characteristic shown on the right. Ignore leakage fluxes and fringing effects in this problem.

<table>
<thead>
<tr>
<th>( N ) turns</th>
<th>( l_1 ) cm</th>
<th>( l_2 ) cm</th>
<th>( A_1 ) cm(^2)</th>
<th>( A_2 ) cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>80</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Given the above data, find the current \( I \) required to establish a flux density \( B_1 = 0.5 \) T in the portion with the larger cross-section.

(b) How much magnetic energy \( W_m \) (J) is stored in the magnetic structure for the condition of Part (a)?

(c) What is the maximum magnetic energy \( W_m \) (J) that can be stored in this magnetic structure?
Two coils are wound as shown on a magnetic core with infinite permeability. Neglect leakage fluxes and fringing effects in this problem.

These two coils are characterized by the following flux-current relationships:

\[
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
\]

(a) Add polarity marks to the mmf’s in the equivalent magnetic circuit shown above.
(b) Express the self inductances \( L_{11} \) and \( L_{22} \) of the two coils in terms of \( N_1, N_2, \) and \( R_g \).

(c) Express the mutual inductance \( L_{12} = L_{21} \) of the two coils in terms of \( N_1, N_2, \) and \( R_g \), and specify its sign.

(d) Find \( v_2(t) \) if \( N_1 = 2N_2, v_1(t) = 100 \cos 120\pi t \) V, and \( i_2(t) = 0 \).